كينماتيكا الهندسة التفاضلية لعديدات الطيات المسطرة في الفضاء الإقليدي ذي الثلاثة أبعاد

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Kinematics Differential Geometry of Line Manifolds In Euclidean 3-Space E^3

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قائمة المحتويات

مُستخلص عربــي

تعتبر دراسة الهندسة التفاضلية لعديدات الطيات المسطرة والمولدة بخط مستقيم في الفراغ المعني واحدة من أهم فروع الهندسة التفاضلية وذلك نتيجة لارتباطها المباشر بالعديد من العلوم الهندسية والفيزيائية وغير ذلك من النواحي التطبيقية.

وقد اهتم كثير من الباحثين بهذا الفرع من الهندسة التفاضلية وقاموا بتطوير الخصائص الهندسية لهندسة الخط المستقيم والتي أصبحت أساساً لوصف كينماتيكا الهندسة التفاضلية المولدة بالسطوح المسطرة والسطوح المتآلفة. ولقد تطور عِلم الهندسة التفاضلية الخاص بها بشكل مُستمر مُنذ نهاية القرن التاسع عشر، حيثُ استحدث شتودي التحويل الآتي:

"مجموعة الخطوط المستقيمة في الفراغ الإقليدي الثلاثي تكون في حالة تناظر أحادي مع مجموعة النقاط على سطح كرة الوحدة الازدواجية"

وتهتم هذهِ الدراسة بدراسة كينماتيكا الهندسة التفاضلية لعديدات الطيات المسطرة في الفضاء الإقليدي ذي الثلاث أبعاد وذلك باستخدام هذا التحويل، وللاستفادة من ذلك في برهنة بعض الصيغ المعروفة في نظرية الأسطح في الفضاء الإقليدي الثلاثي وتفسيرها هندسياً، كما تم إعطاء أمثلة تؤكد ما توصلنا إليهِ من نتائج.

وتحتوي هذهِ الرسالة على خمسة أبواب الوصف التفصيلي لها يكون كما يلي:

في الباب الأول تم عرض بعض المبادئ والمفاهيم الأساسية في علم الهندسة التفاضلية والتي تلزم لإتمام الرسالة، حيث قدمنا أولاً وبصورة موجزة بعض المفاهيم المتعلقة بعديد الطيات التفاضلي وزمرة لي وجبر لي وهندسة عديدات الطيات المسطرة في الفضاء الإقليدي الثلاثي، وأخيراً نوضح مفهوم الأعداد والمتجهات الازدواجية وشرح مبدأ شتودي للتحويل والمصفوفات الازدواجية.

وفي الباب الثاني تم إعطاء العلاقات المصفوفية للحركة الإقليدية العامة والحركة الكروية الازدواجية، كما تم إيجاد العلاقة المصفوفية بين مراكز الدوران الكروية الازدواجية اللحظيــــة. وأخيراً تم استنتاج الصيغة الازدواجية لصيغة كيلي للكينماتيكا الكروية وتفسيرها هندسياً. ومن ثم تم إيجاد مُتجه السرعة الزاوي الازدواجي وتم استنتاج بعض النظريات.

وفي الباب الثالث قمنا باشتقاق معادلات السطوح المسطرة الثابتة والمتحركة المولـــدة بواسطة محور الدوران اللحظي في الحركة الإقليدية العامة ذات المتغير الواحد والعلاقة بينـــهما. ومن ثّم قدمنا الصيغة الازدواجية لصيغة أويلر-رودرجوز. وأخيراً، تم إيجاد الخصائص الهندســـية الكينماتيكيه لتلك السطوح وشرحها تفصيلياً.

وفي الباب الرابع قمنا بدراسة حركة بلاشكا كتطبيق على نظرية الحركة العامة والتي تكون فيها السطوح المولدة بمحور الدوران اللحظي ومنحنيات الانكماش مرتبطة صراحة،حيث تم إيجاد بعض الصيغ الجديدة التي تصف العلاقة الهندسية بينهما. وكذلك تم الحصـــول علـــى الخصائص الكينماتيكية في شكل بسيط. ثم بينا أنهُ بالنسبة لحركة بلاشكا فإن محور الـــدوران اللحظي يولد سطح بولكر. وأخيراً قد تم إيجاد بعض النتائج التي تصف السطح المتآلف الزائدي وتم مناقشة حركة فرينيه كحالة خاصة من حركة بلاشكا.

وفي الباب الخامس وكتطبيق على مبدأ شتودي للتحويل ولتفسير بعض النتائج تم تقديم الحركة الإقليدية العامة ذات المتغير الواحد لمسارات الخطوط المستقيمة المرتبطة بالفضاء المتحرك على أنها سطح مسطر في الفضاء الثابت وبالتالي تأكيد ما حصلنا عليهِ من نتائج.

Abstract

The theory of line geometry was first introduced by Plücker who published a book on the subject: Details concerning line geometry and its applications can be found for instance in the works of Blaschke, W. [6], and Bottema, O. & Roth, B. [6]. Pottman, H. & Wallner, J. [18] developed the differential properties of line geometry that became the basis for describing the differential geometry of the kinematically generated line congruence and ruled surfaces. The differential line geometry is continuously developing, with some of the most influential works, see for example [1-4, 14-21]. In our times, methods and techniques have changed and developed; kinematics is still a field of research for mathematicians. Line geometry turned out to be important for understanding of spatial kinematics especially ruled surfaces have a meaning in practical applications.

The present work is deal with a new technique for describing and distinguishing kinematics-differential geometry of line trajectories in spatial motions. Different generic elements in a moving rigid body generate different trajectories. The trajectory of a point in a moving rigid body is in general, a space curve, while the trajectory of a line in a moving rigid body generates a ruled surface. In kinematics-geometry, the main emphasis is on the study of the invariants properties of the line trajectories of a moving rigid body. The relationships between line geometry and kinematics mad it is necessary to investigate and study line trajectories in spatial motions. This thesis consists of five chapters. A detailed description of these chapters would run as follows:

In chapter 1, firstly for later use, we give some formulae concerning of differentiable manifold, Lie groups & Lie algebra, and line geometry in three-dimensional Euclidean space, as introduced by Refs.[16-25], are presented. Finally, the E. Study's map and the elements of dual matrices as well as the relationships with real matrices representations are formulated.

In chapter 2, In this chapter the matrices relationships concerned with spatial Euclidean motion and dual spherical motion are presented. As well as a relation matrix between the instantaneous dual spherical centers of rotations has been formulated.

In chapter 3, an algebraic approach for deriving the equations of axodes for one-parameter spatial Euclidean motion has been introduced. Then the dual versions of the well-known formulae of Euler-Rodrigues and Cayley of spatial kinematics are derived and explained. Finally, the geometrical- kinematics properties of the axodes are investigated and examined in detail.

In chapter 4, we study the Blaschke's motion furnishes an illustration of the general theory wherein the axodes and their striction curves can be determined explicitly and yet are not trivial. In terms of this, some new formula which can easily give a clear insight for description their relationship. Then, it is shown that for the Blaschke's motion the trajectories of the instantaneous screw axis (ISA) constitute the Plücker's conoid. Finally, the results are carried out that describing an hyperbolic line congruence, and as a special case the Frenet's motion is discussed.

In chapter 5, we demonstrate the use of dual vectors and E. Study's map for elucidate the proof of some results. Then, some examples of applications are introduced and investigated in detail.

SUMMRY

The theory of line geometry was first introduced by Plücker who published a book on the subject: Details concerning line geometry and its applications can be found for instance in the works of Blaschke, W. [6], and Bottema, O. & Roth, B. [6]. Pottman, H. & Wallner, J. [18] developed the differential properties of line geometry that became the basis for describing the differential geometry of the kinematically generated line congruence and ruled surfaces. The differential line geometry is continuously developing, with some of the most influential works, see for example [1-4, 14-20]. In our times ruled surfaces do not have become more important than the last two centuries. But now, methods and techniques have changed and developed; kinematics is still a field of research for mathematicians. Line geometry turned out to be important for understanding of spatial kinematics especially ruled surfaces have a meaning in practical applications.

The study of line trajectories of general rigid body motion consists of two parts: the orientation and location of the moving line. The orientation of the moving line is represented by a cone. The intersection of the cone with a unit sphere centered at the apex defines a spherical curve which is known as the spherical indicatrix or the spherical image of the line trajectory. The location of the moving line with respect to a reference point is defined by a space curve known as the directrix of the line trajectory.

As it is will known, an important analytical tool in the study of three-dimensional spatial kinematics and the differential line geometry is based upon dual vector calculus numbers as shown in [17-19]. The dual

number is used to recast the point displacement relationship into relationships of lines. The dual numbers were first introduced by Clifford [9] after him Study, E [25] used it as a tool for his research on the differential line geometry. A more recent description of this representation can be found in the works [1-4, 16-20].

The present work is deal with a new technique for describing and distinguishing kinematics-differential geometry of line trajectories in spatial motions. Different generic elements in a moving rigid body generate different trajectories. The trajectory of a point in a moving rigid body is in general, a space curve, while the trajectory of a line in a moving rigid body generates a ruled surface. In kinematics-geometry, the main emphasis is on the study of the invariants properties of the line trajectories of a moving rigid body. The relationships between line geometry and kinematics mad it is necessary to investigate and study line trajectories in spatial motions. This thesis consists of five chapters. A detailed description of these chapters would run as follows:

In chapter 1, firstly for later use, we give some formulae concerning of differentiable manifold, Lie groups & Lie algebras, and line geometry in three-dimensional Euclidean space, as introduced by Refs.[16-24], are presented. Finally, the E. Study's map and the elements of dual matrices as well as the relationships with real matrices representations are formulated.

In chapter 2, In this chapter the matrices relationships concerned with real spatial motion and dual spherical motion are presented. To study the geometrical properties of the motions, it is shown that the E. Study's map is linear isomorphisim mapping. Then, some results and theorems are introduced, as well as a relation matrix between the instantaneous dual spherical centers of rotations has been formulated.

In chapter 3, an algebraic approach for deriving the equations of axodes for one-parameter spatial Euclidean motion has been introduced. Then the dual versions of the well-known formulae of Euler-Rodrigues and Cayley of spatial kinematics are derived and explained. Finally, the geometrical- kinematics properties of the axodes are investigated and examined in detail.

In chapter 4, we study the Blaschke's motion furnishes an illustration of the general theory wherein the axodes and their striction curves can be determined explicitly and yet are not trivial. In terms of this, some new formulae which can easily give a clear insight for description their relationship which concerning with their geometrical and kinematics properties are expressed in simple form. Then, it is shown that for the Blaschke's motion the trajectories of the instantaneous screw axis (ISA) constitute the Plücker's conoid. Finally, the results are carried out that describing an hyperbolic line congruence, and as a special case the Frenet's motion is discussed.

In chapter 5, we demonstrate the use of dual vectors and E. Study's map for elucidate the proof of some results. As it is well known that during the one-parameter Euclidean spatial motion, the trajectories of an fixed oriented line, belonging to moving space, is generally ruled surface in a fixed space H_f. Then, some examples of applications are introduced and investigated in detail.

(لايوجد ملخص عربي ونتائج)